Philosophy of Logic
An Anthology

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The Laws of Logic

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Tautologies and Analytic Statements

The statement that arithmetic is, with the help of adequate definitions, reducible to logic hardly clarifies the nature of arithmetic as long as we do not know what a logical truth is. The philosophers of science who have contributed the lion's share to the clarification of mathematics, logic, and the relation of these formal sciences to experience, are the logical positivists. And it is one of their characteristic tenets that the laws or truths of logic are tautologies and thus have no "factual content"; another terminology often used to make the same claim is that they are "analytic," in contrast to the synthetic propositions established by the factual sciences. It is also not uncommon to pass from this assertion to the conclusion that the allegedly inexorable necessity of the laws of logic is somehow reducible to linguistic conventions. The latter thesis is sometimes called *logical conventionalism.* In order to make up our minds about its merits, we must first attend carefully to the meanings of the key terms in this important controversy in the philosophy of science – "tautology" and "analytic."

The simple prototype of a tautology is any statement of the form "*p* or not *p*," where "*p*" represents a statement; such as "that man is a banker, or else he isn't." Such a statement is obviously true regardless of whether its component statement "*p*" is true or false. For a disjunction, i.e., a statement of the form "*p* or *q*," is true provided at least one component statement is true. So, if "*p*" is true, "*p* or not *p*" is true; and if "*p*" is false, then by the very meaning of "not," "not-*p*" is true, so again the disjunction is true. Generalizing from this simple example, we call a tautology any compound statement that is true regardless of whether its component statements be true or false. Sometimes the computation that establishes that a given compound statement is a tautology is quite complicated, just as it may be complicated to prove that a mathematical equation reduces to an identity. The appropriate methods of computation are explained in numerous textbooks of symbolic logic; hence they need not be explained here in detail. Let us just illustrate this concept of tautology in terms of a slightly more complicated example: If *p*, and (if *p*, then *q*), then *q*. This statement form corresponds to the principle of deduction, "Whatever statement is implied by true statements, is itself true." In order to show that any compound statement of that form must be true, regardless of the truth-values of the statements replacing the variables "*p*" and "*q*," we show that it could not possibly be false. In order for it to be false, the antecedent "*p* and (if *p*, then *q*)" – which corresponds to the two premises of a deductive argument of that form – would have to be true while the consequent "*q*" is false. But in order for the antecedent to be true "*p*" must be true, and "if *p*, then *q*" must be true. But if "*p*" is true and "*q*" false, then "if *p*, then *q*" cannot be true. In other words, if the consequent is false, the antecedent is

Arthur Pap, The Laws of Logic. *An Introduction to the Philosophy of Science*. New York: Free Press, 1962. 94–106.

bound to be false – which is equivalent to saying that if the antecedent is true, the consequent is bound to be true.

By contrast, consider: If q, and (if p, then q), then p. It is possible for a true implication to have a false antecedent, hence it is possible that q and (if p, then q) be true and p be false. Hence it is possible for the above complex implication to be false, hence it is not a tautology. Again, "If not-q and (if p, then q), then not-p" is a tautology because in order for the implication to be false p would have to be true while q is false (i.e., not-q true) and (if p, then q) true, which is impossible. It may be left to the reader to apply the same method to establish that "If not-p, and (if p, then q), then not-q" is *not* a tautology.

These illustrations bring out an important connection between the concept *tautology* and the concept *valid deductive argument:* suppose that an implication "If P_1 and P_2 and ... and P_n, then C" is a tautology in the explained sense; then an argument (or inference) whose premises are $P_1, P_2, \ldots, P_n$ and whose conclusion is C, is valid, in the sense that C must be true if $P_1, P_2, \ldots, P_n$ are all true. The converse of this conditional statement, however, does not hold. Thus, consider a valid syllogism, say, one of the form "All M are P, all S are M, therefore all S are P." Here the corresponding implication is not a tautology. For, since the three statements here do not have statements, but rather terms, as parts, each is to be represented by a simple statement variable and the corresponding implication has the form: If p and q, then r. And clearly not all implications of this form are true. Generally speaking, the tautological character of a compound statement depends upon the meanings of the logical connectives, which are particles used to form compound statements out of statements that do not contain statements as parts: "not," "if-then," "or," "and" (and certain others that are definable on the basis of these). The validity of a syllogism, as well as of many other forms of deductive inference, does not, however, depend just on the meanings of logical connectives. Here the logical constants whose meanings are decisive are "all," "some," and "are" as used to express inclusion of one class in another.

In the light of this distinction, the statement that the laws of logic are simply tautologies must be condemned as either false or trivial. It is false if "tautology" is meant in the restricted sense explained above. Let us add an example of a law of logic that is not a tautology in the restricted sense, and which moreover is beyond the scope of Aristotelian logic, which dealt exclusively with syllogistic reasoning: Whatever relation R may be, if something has R to everything else, then everything has the converse of R to something or other (where the converse of R is that relation which x has to y if and only if y has R to x). However, since all tautologies are formal truths, and it is often uncritically assumed that all formal truths are tautologies, the word, "tautology" has also come to be used in the broader sense of "formal truths." But since "formal truth" in any precise sense turns out to be synonymous with "law of logic" (or "logical truth") it is then trivial to say that all laws of logic are tautologies. A "formal truth" or a "law of logic" is a statement that is true by virtue of its logical form, and this means that its truth depends only on the meanings of the *logical constants* it contains, not on the meanings of the *descriptive* terms. "John is tall or John is not tall": clearly you can replace "John" by "Plato" and "tall" by "fat," and the resulting statement will be just as true. Similarly, any statement of the form "If all M are P and some S are M, then some S are P" is true, regardless of what descriptive terms may be substituted for "M," "P," and "S" (provided, of course, that no equivocation is committed). What is usually called a "law of logic" is a purely abstract, universal statement devoid of descriptive constants, such as: For all classes M, P, and S, if all M are P and some S are M, then some S are P. A substitution instance of a law of logic, such as "if all African–Americans have black skin and some Americans are African–Americans, then some Americans have black skin," is called a *logically true* statement.

Teachers of elementary logic explain to their students that one of the most frequent sources of fallacious reasoning is illicit conversion: from all A are B it does not follow that all B are A. Although philosophers are supposed to know elementary logic, they too sometimes commit illicit conversions. Passing from "all tautologies are formal truths, and hence logical truths," to "All logical truths are tautologies" is one illustration. Another instance of illicit conversion is the argument leading from the valid premise "all logical truths are true by sole virtue of the meanings of their constituent terms (in particular of the logical constants)" to the thesis that all statements that are true just by virtue of the meanings of the terms (hence, that do not require empirical verification and that cannot be empirically falsified) are logical truths. Obviously, "All bachelors are unmarried" is true by virtue of the meaning of "Bachelor," but since the word "bachelor" does not belong to the vocabulary of logic, the statement is not logically true. Now, this counterinstance is relatively trivial, since with the help of the definition "A bachelor is an unmarried man" (which fairly accurately expresses the meaning of "bachelor" in English) the above statement is translatable into a logically true statement: All unmarried men are unmarried. Such statements, which are translatable into logically true statements with the help of definitions that express the ordinary meanings of the defined terms, we call *strictly analytic*.

But not all statements that are commonly accepted as necessary, or a priori, truths, are strictly analytic. For example: No event precedes itself. That this is not a logical truth is evident from the fact that the verb "to precede" does not belong to the vocabulary of logic – it designates something we find in the world, a temporal relation – and yet it occurs essentially in the statement, i.e., you cannot substitute in the universal statement "For any event x, x does not precede x" any other grammatically admissible expression for "precede" without changing the truth-value of the statement. For example, if we substitute "occur at the same place as" we obtain a plain falsehood. In other words, the statement has the form: For any x, *not-*(xRx), and since some statements of that form are false, the statement in question is not a formal, or logical, truth. Now, it yet might be strictly analytic. It would be strictly analytic if it were possible to analyze the familiar relation of temporal precedence in such a way that the statement could be translated into a logically true statement – in somewhat the way in which "All uncles have siblings" can be revealed as analytic by the definition "An uncle is a human male who has a sibling who is a parent. "But precedence seems to be a simple relation that does not admit of further analysis. And if so, we have here a *synthetic* statement that is necessarily true (a priori). It has empirical content, if you wish, in the sense that it is about an empirically given relation – unlike logical truths, which do not contain descriptive terms essentially, and in that sense are not "about" the world of experience. But it is not an empirical statement, because an empirical statement, as we use this term, is a statement whose truth or falsehood depends on facts of experience. To be sure, a being who never experienced temporal succession could not possibly understand the meaning of such words as "before," "earlier," "to precede." It does not follow that the assertion we make about this nonlogical relation when we say that it is irreflexive and asymmetrical (and, for that matter, transitive) is subject to the test of experience.

Tautologies and Linguistic Conventions

We have seen that the claim of logical positivism, which derives from Wittgenstein, that systems of logic are systems of tautologies, must be taken with at least one grain of salt. But let us assume, for the sake of argument, that all the laws of logic, including those of the so-called theory of quantification, are in some way reducible to tautologies, as was believed by Wittgenstein. Why did this seem to have great philosophical significance to the logical positivists, as well as to some of their critics? Because it was believed that a tautology owes its *necessity* to the force of linguistic conventions, and that therefore such a reduction would *explain* logical necessity without any metaphysical assumptions. Consider again the prototype of tautology, "p or not-p," which corresponds to the law of the excluded middle: For any proposition p, either p or the negation of p is true. That any statement of this form must be true follows from the definitions of "or" and "not," given in the form of statements of the truth-conditions of disjunctions and negations. Similarly, the principle of deductive inference, that whatever proposition is implied by true proposition is itself true, would seem to owe its validity to the very rule governing the use of "implies": to say that p implies q though p is true and q false, is just as self-contradictory as to say that X is a bachelor and married at the same time. If so, it looks as though the compulsion we feel to assent to these laws of logic is simply the ingrained habit of abiding by the linguistic conventions we were educated to conform to when we were taught the language. But linguistic conventions, after all, *may* be changed. Therefore, say the logical conventionalists, systems of logic may be changed; there is no absolute logical necessity; the logical necessity of a proposition is entirely relative to linguistic conventions, which it is possible to change.

The test of tautology by means of truth tables consists in computing the truth-values of the statement-form in question corresponding to all possible combinations of truth-values of the elementary statements. If and only if the statement-form comes out true in all cases, then it (or its substitution instances) is a tautology. But the outcome of the computation depends, of course, on the definitions of the connectives, such as "or," "and," "if, then." Thus, having laid down for "or," "and," "if, then" the truth-conditions tabulated in tables 1.1, 1.2 and 1.3, we find in table 1.4 that "If [*p* and (if *p*, then *q*)], then *q*" is a tautology but suppose

Table 1.1

p	*q*	*p* or *q*
T	T	T
T	F	T
F	T	T
F	F	F

Table 1.2

p	*q*	*p* and *q*
T	T	T
T	F	F
F	T	F
F	F	F

*Table 1.3**

p	*q*	If *p*, then *q*
T	T	T
T	F	F
F	T	T
F	F	T

*This table defines the so-called material, or truth-functional, meaning of "if, then."

Table 1.4

p	*q*	*p* and (if *p*, then *q*)	*If* [*p* and (if *p*, then *q*)], then *q*
T	T	T	T
T	F	F	T
F	T	F	T
F	F	F	T

we defined "If *p*, then *q*" as a compound statement that is true if *p* is false and *q* is true, and false in all other cases. On the basis of this definition, "If [*p*, and (if *p*, then *q*)], then *q*" would not express a tautology, as shown by table 1.5. More obviously still, if we defined "not" as

Table 1.5

p	*q*	If *p*, then *q*	*p* and (if *p*, then *q*)	If [*p* and (if *p*, then *q*)], then *q*
T	T	F	F	T
T	F	F	F	F
F	T	T	F	T
F	F	F	F	F

signifying a contrary, not the contradictory, of the proposition on which it operates, the law of the excluded middle would cease to be a tautology: since contrary propositions (such as "All alcoholics are unhappy" and "No alcoholics are unhappy") may both be false, "*p*" and "not-*p*" may now both be false, in which case "*p* or not-*p*" would likewise be false. And so on, for any law of logic that might be cited.

If by saying that no law of logic has *absolute* validity we mean that whether or not a given formula or sentence expresses a law of logic (in either the narrower sense, viz. truth-functional tautology, or the broader sense of "formal truth") depends on the interpretation of the logical constants, the claim is undoubtedly correct. But once it is clearly understood that the truth or falsehood of *any* sentence depends on its interpretation, such a "relativism" appears to be quite innocuous. At any rate, in this light the controversy between the conventionalist and the rationalist regarding the necessity of the laws of logic appears rather futile. What one ascribes truth to, be it formal or empirical truth, is never a bare sentence (string of marks, or sequence of noises), but a statement that is made by means of a given sentence, and what that statement is depends on the *meanings* that are assigned to the constituent symbols. Clearly the truth of a statement I am making by the use of a sentence *p* cannot be converted into falsehood by putting upon some symbol contained in *p* an interpretation different from the one I intended. And this is the case whether the statement be necessary or contingent. What I mean by saying "There are no squares that are not equilateral" is necessarily true and will remain so even if the word "square" should come to be used in the sense in which "triangle" is used at present. If at such a later time, at which we are supposing the relevant linguistic conventions to be different, the same words were used in accordance with what were then the linguistic conventions, they would be used to make a false statement. But that does not mean that the statement I am *now* making by means of that sentence would have been falsified.

It is hard to believe that the conventionalist interpretation of the laws of logic, which has been advocated by acute, sophisticated philosophers, amounts to just a gross failure to distinguish between a bare sentence (a certain kind of sequence of linguistic signs) and an assertion made by means of a sentence. Some conventionalists have meant to say, indeed have said explicitly, that the rationalists err in regarding the traditional laws of logic as necessary truths apprehended by reason, because they are "laws" in a prescriptive rather than a descriptive sense. When we speak of the laws of nature, such as the law of freely falling bodies, the law of gravitation, the laws of chemistry, we mean universal statements that *describe* the world, the course of nature as it happens to be. Now, the laws of logic do not describe any contingent features of the world that can be conceived to be different. They do not even describe mental phenomena, e.g., men's habits of drawing such and such conclusions from such and such premises. For if we find a man reasoning fallaciously, i.e., inferring from propositions assumed to be true a proposition that just does not follow from them, we do not say that the relevant law of logic has been refuted. We are prepared to describe conceivable observations that would refute certain presumed laws of nature, including laws of mental association, but it would be even absurd to suppose that any observations, whether of physical or of psychological facts, might ever refute a law such as "If a thing has either property P or property Q, and it does not have P, then it has Q." According to the conventionalist's diagnosis of rationalism, the rationalist has been led to postulate a mysterious realm of necessary truths apprehended by reason because, while realizing that the valid sentences of logic do not describe empirical facts, he makes the mistaken assumption that they do describe facts of some kind. But, says the conventionalist, they are not descriptive sentences at all, they are *rules*. In particular, they are rules for the use of logical constants. Naturally, a rule cannot be refuted by any facts, because it does not make sense to speak of "refuting" a rule; a rule can only be violated.

In order to understand this conception of laws of logic as linguistic rules, we should reflect on the method of specifying the meanings of logical constants, i.e., such expressions as "and," "or," "not," "if, then," "all," which are involved in scientific discourse about any subject-matter. The validity of a statement of logic depends only on the meanings of logical constants, but how are the latter to be specified? Explicit definition is not possible. Some logical constants, on the other hand, can be contextually defined in terms of others. Examples:

all things have property P = *not*-(*some* things do *not* have P)
p *and* q = *not*-(*not*-p *or* *not*-q)
p *or* q = *if* *not*-p, *then* q

Let us assume that in our logical system the logical constants here used to define contextually "all," "and," and "or," viz. "some," "not," and "if, then," occur as primitives. How are we to explain their meanings, their rules of usage? Superficially it seems that this can easily be done (at least for "not" and "if, then") by means of truth tables which stipulate the conditions under which statements of the forms "not-p" and "if p, then q" are true. The truth table for "not" is very simple:

p	not-p
T	F
F	T

Here "T" means true, and "F" false, and the table is to be read from left to right as follows: if p is true, then not-p is false, and if p is false, then not-p is true. But as a definition this is circular if "p is false" is in turn defined as "p is not true." More obviously still, it would be circular to attempt to explain the meaning of "if, then" by means of a truth table. Quite apart from the consideration that the truth of a conditional statement (i.e., statement of the form "If p, then q") does not just depend on the truth-values of the component statements, but rather on their meanings [technically this is expressed by saying that "if, then" is, in most uses, not a truth-functional connective], it is clear that we use "if, then" in interpreting any truth table. For a truth table says that a given kind of compound statement, such as conjunction, disjunction, negation, is true *if* the combinations of truth-values of the component statements are such and such. We must have recourse, then, to another method of formulating the rules of use of the primitive logical constants. The method in question differs fundamentally from *definition* in the usual sense, i.e., formulation of rules of substitution or translation by virtue of which the defined expression is theoretically eliminable. It is the method of *postulates*.

Thus we might explain "if, then" by stipulating that all statements of the following forms are to be true (note that this is different from *asserting* that all such statements *are* true, for according to ordinary usage of "assert," "I assert that p is true" makes sense only if it makes sense to doubt whether p is true, but such doubt is senseless if "p" just serves to specify, partially, the meanings of constituent terms):[1] if (if p, then q), then, if (if q, then r), then (if p, then r); if p, then, if (if p, then q), then q. Then we might add postulates introducing "not" along with "if, then": if p, then not-(not-p); if not-q, then, if (if p, then q), then not-p; if p, then (if not-p, then q). We have postulated, then, that all statements derivable from these schemas by substituting statements for the statement variables (in such a way that the same statement replaces the same variable within a given schema, though the same statement may be substituted for different variables) are to be true. The schemas correspond to the following principles of logic: the principle of the hypothetical syllogism (corresponding to *barbara* in the theory of categorical syllogisms); a statement implied by a true statement is true (*modus ponens*); the principle of noncontradiction; a statement that implies a false statement is false (*modus tollens*); from a contradiction any proposition follows.

The conventionalist, now, maintains that it is senseless to speak, in the manner of rationalists, of insight into the necessary truth of such principles, because they are nothing but conventional assignments of meanings to the logical constants "if, then" and "not." It does not make sense to ask how we know, indeed know for certain, that every substitution instance of these schemas is true, because no cognitive claim is involved in stipulations of rules of usage. You can say, "I do not wish to use 'if, then' in such a way that every substitution instance of this schema is true," but it would be nonsense to say, "I do not believe that all substitution instances of this schema are true." In the same way, if one were to stipulate, " 'Green' is to be used to designate the color of these objects," he might be opposed by one who, for whatever reason, did not wish to use the word "green" that way. But one cannot sensibly counter: "Before accepting your rule I want to make sure that those objects really are green."

To be sure, if the expression for which a rule of usage is laid down already has a prior use, one can sensibly ask whether the rule conforms to that prior use. In the case of our logical schemata, it is clear that if any logician were to "postulate" them (in the explained sense), he would be guided by his familiarity with the already existing rules of usage of the logical constants. He would not, for example, postulate that all substitution instances of "If *q*, then (if *p*, then *q*), then *p*" are to be true, because if he did, he would require us to use "if, then" differently from the way it is in fact used. In other words, according to the actual use of "if, then" in English not all substitution instances of this schema are true. Whether or not the stipulations accord with actual linguistic usage is a question of empirical fact. But what the conventionalist is out to refute is the view that our knowledge of logical truths amounts to a priori knowledge of necessary propositions. Our knowledge that, say, the logical constants "if, then" and "not" are so used by English-speaking people that all substitution instances of, say, "If *p*, then not-(not-*p*)" are true, is just plain empirical knowledge. It is, of course, conceivable that a man might deny a statement of that form, but in that case we would just have to conclude that his speech habits are different: perhaps he uses "if, then" the way "either-or" is ordinarily used, for example. But to tell him "You cannot deny it, *because it is necessarily true*" is, according to the conventionalist, like saying "You must speak the way we speak, because you have to speak that way."

Yet, the conventionalist cannot get around the admission that there is such a thing as a priori knowledge of logical truths, which is in no intelligible sense reducible to stipulation of, or acquaintance with, linguistic rules. In the first place, it is a meaningful question to ask whether it is possible, say, to define "if, then" on the basis of "not" and "or" in such a way that (*a*) the definition accords approximately with ordinary usage, (*b*) our postulates are transformed into truth-functional tautologies if "not" and "or" are defined as truth-functional connectives in the usual way. The definition that fulfills these requirements is: If *p*, then *q* = not-*p* or *q*. We know, for example, on the basis of truth-table analysis, that any statement of the form "not-*p* or not-(not-*p* or *q*) or *q*" (the transform into primitive notation of "if *p*, then, if (if *p*, then *q*), then *q*") is a tautology. Surely it does not make sense to say *it is a linguistic rule* that in a language containing the mentioned rules for the use of "not" and "or" any statement of the above form is a tautology. Indeed, this metastatement is a necessary statement, not a contingent statement about linguistic usage. That is, it is inconceivable that, while the rules for the use of "not" and "or" remain the same, a statement of the above form should fail to be a tautology.

Secondly, logicians usually lay down their postulates, not in order to prescribe a usage for logical constants or to describe how they are in fact used, but in order to construct a system, and this means that they intend to deduce a lot of theorems from the postulates. These deductions are, of course, guided by rules of deduction. Two of the most important rules of deduction (whether or not they be absolutely indispensable) are the rule of substitution and the rule of detachment (or "modus ponens"). The rule of substitution says with reference to our postulates: any formula obtainable from a postulate by substituting for a statement variable another statement variable or a truth-function of a statement variable, the same substitution being made for each occurrence of a given variable, is a theorem (and any formula derivable from a theorem in the same manner is also a theorem). The rule of detachment says: if A and (if A, then B) are postulates or theorems, then B is a theorem (here A and B are syntactic variables ranging over formulae of the system). Without raising the question of the justification of these rules of deductive proof, we wish to insist on the following simple point: a metastatement to the effect that such and such a formula is a theorem in the system that is characterized by such and such postulates and such and such rules of deduction is not a "rule" of any kind. It is, if true, *necessarily true*. It is a fact that cannot be altered by changing rules, that in a deductive system with specified formation rules, postulates, and rules of deduction, such and such a formula is a theorem whose proof involves such and such a minimal number of elementary steps. That the discovery of such "facts" by mathematicians and logicians involves the manipulation of symbols in accordance with rules is entirely consistent with its being an intellectual discovery – even if it is a proposition about symbols and not about intangible and invisible abstract entities. Even if algebra were construed as a science whose subject-matter consists of symbols, not of abstract entities such as numbers, it would be a meaningful question whether, say, Fermat's "last theorem" (for $n > 2$, there are no solutions for the equation: $x^n + y^n = z^n$) is really a theorem in such and such a system of algebra. Mathematicians have not found the answer yet, but most of them regard it as a serious and meaningful question. And the proposition in question is either necessary or impossible; it is not an empirical proposition. It would be silly to say that the question here is whether such and such rules ought to be adopted. The question is not like the question whether there is a number that satisfies the equation "$x^2 = 2$"; it is rather like the question whether there is a rational number that satisfies that equation. It was, indeed, no discovery that there is an irrational number that satisfies it. This was a matter of decision, of deciding to broaden, by fiat, the extension of the term "number," whatever the reasons motivating the decision may have been. But Euclid did not *stipulate* that the equation has no rational solution; he *discovered* it by a well-known indirect proof.

We conclude that though logical conventionalists have rendered a valuable service in focusing attention on the role played by linguistic conventions in the acquisition of logical and mathematical knowledge, they have not shown that there is no such thing as a priori knowledge of necessary propositions and that the necessity of the laws of logic "depends" in some intelligible sense on linguistic conventions. In particular, to say of a certain complicated statement that it is a tautology, is not to deny that it is necessarily true nor that it makes sense to speak of "discovering" its truth; it is rather to explicate what the necessity and its discovery consist in. The thesis of Whitehead and Russell that all mathematical propositions are tautologies is still acutely controversial; and the thesis that all necessary propositions are tautologies is certainly false. But whether it be true or false has no bearing whatever on the question whether there is such a thing as purely intellectual discovery of necessary truths. Of course there is such discovery. And the discovery by means of some mechanical decision procedure (such as the use of "truth tables") that a certain complicated form of deductive argument is valid because the corresponding implication is a tautology is not the least useful and respectable among such intellectual discoveries.

Note

1 Readers who are untrained in formal logic will find it easier to grasp the sense of these postulates if they occasionally replace "if p, then q" by "p implies q" – though this is technically inaccurate inasmuch as grammar requires "p" and "q" to be quoted when they are connected by "implies." The first postulate, for example, is then recognizable as the principle of the hypothetical syllogism in the form: if "p" implies "q," then, if "q" implies "r," then "p" implies "r."